Getting Started with the *CandyFactory* Educational Game

So, you've downloaded *CandyFactory* from the iTunes store and are looking for ways it might help your students. Well, this guide is intended to help *you*. It includes the following four components.

- LEVEL DESCRIPTIONS (pages 2-3): The guide includes descriptions for each of the game's five levels, which support students' progression toward more and more sophisticated conceptions of fractions. In particular, for many students, their only concept for 3/5 is three out of five. Level 1 of *CandyFactory* leverages this concept to get students started. Higher levels push students to think about 3/5 as a measure or size: A fraction that is 3/5 as big as the whole and three times as big as 1/5. Still higher levels push students to generalize this concept to improper fractions and to reason in reverse (producing the whole from a given fraction of it).

- CCSSM ACTIVITIES (pages 4-11): The guide also includes a list of related Common Core State Standards for Mathematics (CCSSM) and three related activities. The first activity addresses fraction comparisons; the second addresses equivalent fractions; and the third addresses fraction multiplication.

- PROMPTS (page 12): Next, we have included several reflective questions that teachers might pose to students during and after game play. These questions are intended to help students organize what they have learned from playing *CandyFactory*.

- REFERENCES (page 13): Finally, we provide three references that discuss the underlying concepts (schemes) *CandyFactory* is intended to support: two NCTM journal articles, from *Teaching Children Mathematics* and *Mathematics Teaching in the Middle School*; and one book.

Also, be sure to check out videos demonstrating game play and learning objectives, posted on our project web site: http://ltrg.centers.vt.edu/

We hope your students enjoy the experience of playing and learning in the Candy Factory.

Sincerely,

The *Learning Transformation Research Group* at Virginia Tech

CandyFactory Levels and the Fractions Learning Progression

Level 1
Partitions are visible in the whole and the customer order, which is always a proper fraction (*m/n*, where *m* is less than *n*). So, students can get oriented to the game while relying on only part-whole concepts (*m/n* as *m* parts out of *n* equal parts in the whole). They can count the number of equal pieces in the customer order (*m*) and the number of equal pieces in the whole (*n*) to determine the fraction is *m/n*.

Steffe and Olive (2010) refer to this way of operating as the *part-whole fraction scheme*, and it is the first scheme students construct for conceptualizing fractions. This is a common way of conceptualizing fractions, but very limited, especially when students begin to consider improper fractions - note that *m* out of *n* makes no sense when *m* is greater than *n*.

Level 2
Partitions are no longer visible in the whole, but the customer order is always a unit fraction (1/*n*). At this level, students can practice slicing the whole with finger swipes, and they begin to understand 1/*n* as the unit fraction that fits into the whole **n** times. In other words, they begin to treat 1/*n* as the **unit** that can be iterated *n* times to make the whole.

Treating unit fractions as iterable units marks the first level of progress in transcending part-whole conceptions of fractions. Students begin to understand unit fractions as sizes relative to the whole by determining how many times the unit fractional piece iterates within the whole. Steffe and Olive (2010) refer to this way of operating as the *partitive unit fraction scheme*.

Level 3
This level is like Level 2, except now the customer order can be *any* proper fraction. Students should begin to understand *m/n* as *m* copies of 1/*n* and as a size relative to the whole.

This way of operating is a generalization of the partitive unit fraction scheme, which Steffe and Olive (2010) refer to as the *partitive fraction scheme*.

Level 4
This level is like Level 3, except now the customer order can be any fraction, including *improper* fractions (*m/n*, where *m* can be greater than *n*).

Unless the student can coordinate all of the units involved, the whole is lost within the improper fraction. The student needs to coordinate the unit fraction, the improper fraction, and the whole within the improper fraction, because the improper fraction exceeds the whole. Students who don't coordinate these three units will often confuse the improper fraction with the whole. Students who can coordinate these three units are said to be operating with an *iterative fraction scheme.*
(Steffe & Olive, 2010).

Level 5

This level is the reverse of Levels 3 and 4. Students are given a fraction (proper or improper) and asked to produce the whole from it. For example, given a piece that is *m/n* of the whole, students need to slice the given piece into *m* parts and make *n* copies of that piece. This way of operating is referred to as the reversible partitive fraction scheme (Steffe & Olive, 2010).

From: Steffe, L. P., & Olive, J. (2010). *Children's Fractional Knowledge*. New York: Springer.

Mapping the Common Core State Standards for Mathematics to *CandyFactory*

Grade 4

- **Extend understanding of fraction equivalence and ordering.**
  - CCSS.Math.Content.4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

    At all levels, students can satisfy the customer order by producing an equivalent fraction. For example, the game will accept 4/6 as an appropriate production for a customer order of 2/3. Teachers can take advantage of this by asking students to think of all of the ways to produce a customer order once one answer is found (see Activity 2).

  - CCSS.Math.Content.4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.

    Throughout CandyFactory, students are adjusting the denominator (slices) and numerator (copies) to create customer orders pertaining to different wholes. By examining the Shift Log, they can visualize how 1/5 of one whole might be larger than 1/2 of another. (See also Activity 1, Extension 1).

- **Build fractions from unit fractions**
  - CCSS.Math.Content.4.NF.B.3 Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.

    The Copy screen helps students understand that a fraction, say 5/6, is just 5 iterations of 1/6. Thus, just as 3+2=5, 2/6+3/6=5/6.

    - CCSS.Math.Content.4.NF.B.3a Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.
    - CCSS.Math.Content.4.NF.B.3b Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8.
    - CCSS.Math.Content.4.NF.B.3c Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
    - CCSS.Math.Content.4.NF.B.3d Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.

- CCSS.Math.Content.4.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction by a whole number.

  Levels 2 through 5 support students' understandings of fractions as size relative to a given whole. In this sense, fraction multiplication, such as 2/3 x 3/5, means 2/3 of 3/5. In other words, students can conceptualize fractions multiplication as 2/3 of the candy bar that is 3/5 of the whole candy bar.

  - CCSS.Math.Content.4.NF.B.4a Understand a fraction a/b as a multiple of $1/b$. CCSS.Math.Content.4.NF.B.4b Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number.
  - CCSS.Math.Content.4.NF.B.4c Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem.

    By using rectangular models in the game, students will also be accustomed to models used to show fractions multiplication and division, as in Activity 3.

- **Understand decimal notation for fractions, and compare decimal fractions.**

  The Copy screen in the game helps students develop a visual model for fractions addition and subtraction, including tenths, and this can be generalized to all decimals.

  - CCSS.Math.Content.4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.

    See Standard 4.NF.B.3 and Activity 2 extension 1.

  - CCSS.Math.Content.4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100.

    See Standard 3.NF.A.3 and Activity 2 extension 1.

  - CCSS.Math.Content.4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.

    See Standard 4.NF.A.2 and Activity 1 extension 3.

Grade 5

- **Apply and extend previous understandings of multiplication and division.**
  - CCSS.Math.Content.5.NF.B.4 Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
    - CCSS.Math.Content.5.NF.B.4a Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b..)

      See Activity 3.

  - CCSS.Math.Content.5.NF.B.5 Interpret multiplication as scaling (resizing), by:

- CCSS.Math.Content.5.NF.B.5a Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.

  As iteration and size comparisons are throughout Candy Factory, this standard is supported in all five levels.

- CCSS.Math.Content.5.NF.B.5b Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.

  Beginning with Level 4 of Candy Factory, students use iteration to make compare measurements between a whole and an improper fraction and a whole and a proper fraction.

- CCSS.Math.Content.5.NF.B.6 Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.

  In Level 5 of CandyFactory, students solve problems by performing partitioning and iterating operations on both proper or improper fractions. See Activity 3.

- CCSS.Math.Content.5.NF.B.7 Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.[1]
  - CCSS.Math.Content.5.NF.B.7a Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3.

    See Activity 3 Extension 1.

  - CCSS.Math.Content.5.NF.B.7b Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4.

    As partitioning and size comparisons are throughout CandyFactory, this standard is supported in all five levels.

  - CCSS.Math.Content.5.NF.B.7c Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem.

    In Level 5 of CandyFactory, students solve problems by performing partitioning and iterating operations on both proper or improper fractions. See Activity 3 Extension 1.

Grade 6

- **Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**
  - CCSS.Math.Content.6.NS.A.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

    See Activity 3 Extension 1.

    CCSS.Math.Content.6.EE.B.8 Write an inequality of the form $x > c$ or $x < c$ to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form $x > c$ or $x <$ c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

  See Activity 2 Extension 2 and Activity 1 Extension 1.

Activity 1 –Estimate and Compare

Supports Standards: CCSS.Math.Content.3.NF.A.3d; 4.NF.A.2; 4.NF.C.7; 6.EE.B.8

Motivation for Student: Sometimes you make the customer order correctly on the first try, but other times we find out that our result was too big or too small when we measure. Sometimes we accidentally try the same slice and copy more than once! What we're going to do today is to record whether our guesses are too big, too small, or just right, so that hopefully we can get them correct in fewer attempts.

Instructional Sequence:

(1) Bring up an example from Candy Factory (any of levels 1-4). Before choosing the candy, explain that they are to quickly look at the customer order and the whole, pause the game, and write down a guess for the fraction the customer order is of the whole.

(2) Ask students to volunteer their guesses after pausing the game. Choose a response that is too small and record it on the board. Then proceed to make that fraction in the game. Ask for student volunteers about how many slices to make, and how many copies to make, but make the estimate correctly. *This will reinforce the relationship between the fraction name and the operations of slicing and copying; i.e., that the number of slices is the denominator and the number of copies is the numerator.*

(3) After measuring, and finding out that the result was too small, record, "too small" next to the estimate and repeat the process with a guess that's too big. Record that fraction next to the first and write "too big." Finally, create and record the correct order.

Example Recording: 2/3 too small
3/3 too big
3/4 just right

(4) Explain that they will repeat this process for the each customer order they make. Challenge them to estimate the correct fraction in as few tries as possible.

(5) At the end of the activity, ask students to write a sentence about which customer orders were the easiest or hardest for them to estimate. Ask volunteers to explain how recording the fractions helped them get correct orders in fewer tries.

Modifications/Extensions:

(1) Students are instructed to use the symbols <, >, or = and a variable name for the customer order, e.g., 2/3 < x instead of 2/3 too small.

(2) Students are instructed to put fractions in sequence from least to greatest after they have found the correct order, e.g., 2/3 < 3/4 < 3/3.

(3) Students are instructed to convert fractional estimates into decimals (or percents, or numbers written in scientific notation, or some combination of the three). For the example, they might write 2/3 ≈ 67% too small; 3/3 = 100% too big; ¾ = 75% just right.

(4) Students are instructed to write fractional estimates as ratios. For the example, they might write 2:3 instead of 2/3.

Activity Two – Equivalent Fractions

Supports Standards CCSS.Math.Content.3.NF.A.3b; 4.NF.A.1; 4.NF.C.5; 4.NF.C.6; 6.EE.B.8

Motivation for Student: Some of you may have noticed that there is sometimes more than one way of slicing and copying to make a customer order. Today we are going to see how many different, but *equivalent* ways we can make a customer order.

Instructional Sequence:

(1) Bring up an example from Candy Factory (any of levels 1-4). After choosing the candy, pause the game and ask students to suggest a number of slices and copies to use. Choose an **incorrect** suggestion. Perform the slices and copies, and get to the measure screen so students can see that their answer was incorrect. Go back to the slice screen.

(2) Make orders based on student suggestions until you find one that is correct. At the measure screen, instead of shipping, record the correct fraction. Then ask volunteers to suggest a way make the customer order a different way. If no one volunteers a fraction, ask a volunteer to suggest a number of slices.

(3) Go back to the slice screen and make the new fraction. At the measure screen, if the new fraction is also correct, then have the students write down the equivalent fraction and put an equal sign between them.

(4) Ask if there are any other ways to make the customer order. Some students might suggest equivalent fractions that cannot be made in the game, such as 9/12. Acknowledge that it is equivalent, but that we are only writing down equivalent fractions that can be made in the game.

(5) Repeat this process until the equivalent fractions have been exhausted. (If the first example you try has no equivalent fraction in the game, do a second example that includes at least one pair of equivalent fractions).
Example Recording:

$$3/4 = 6/8$$

(6) Challenge students to write down every equivalent fraction to a correct customer order that they find.

(7) At the end of the activity, ask students to write a sentence about how they found equivalent fractions. Ask volunteers to explain how they could tell if a customer order could only be made in one way.

Modifications/Extensions:

(1) Students make the correct fraction only once. They then must write this fraction as an equivalent decimal, percent, ratio, or number in scientific notation. Example $3/4 = 0.75 = 75\% = 3$ to 4.

(2) Students record the incorrect fractions they make when attempting additional equivalent fractions, e.g., $3/4 < x$ or $4/5 > x$ for goal x

Activity Three – Fractions Multiplication

Supports Standards CCSS.Math.Content.5.NF.B.4a,b,c; 5.NF.B.4a; 5.NF.B.6; 5.NF.B.7a,c; 6.NS.A.1

Motivation for Student: We've been making a fraction of a whole. What does it mean to make a fraction of a fraction?

Instructional Sequence:

(1) Bring up an example from Candy Factory (any of levels 1 – 4). Ask for student volunteers for the correct number of slices and copies. After making the correct customer order, write on the board a number sentence that reflects the relationship between the customer order and the whole candy bar. Then repeat with the next customer order.
Example Recording:
2/3 of whole is 2/3 . Slice whole into 3 and copy 2 times.
1/4 of whole is 1/4. Slice whole into 4 and copy 1 time.

(2) Question for students: What is 2/3 of 1/4?. Ask volunteers to assist with coming up with the sentence: Slice (1/4) into 3 and copy 2 times. Have students assist with drawing and labeling the following rectangles for the class: the whole, 1/4, and 2/3 of 1/4.

(3) Ask volunteers to find and explain another name for 2/3 of 1/4. *in terms of slicing and copying,* and record the resulting relation.
Example Recording:
2/3 of 1/4 is 1/6. Slice whole (1) into 6 and copy 1 time.

(4) Instruct students to write similar sentences for each consecutive pair of customer orders they encounter in levels 1 – 4.

(5) At the end of the activity, ask volunteers to explain patterns that they found useful. Possibly, solicit students' perception of a connection to the algorithm for fraction multiplication.

Modifications/Extensions:

(1) Extend to division by using Level 5.
Example recording:
(What) of 2/3 is the whole. Slice 2/3 into 2 and copy 3 times.
3/2 of 2/3 is the whole.
Repeat with a second example from Level 5.
(What) of 3/4 is the whole. Slice 3/4 into 3 and copy 4 times.
4/3 of 3/4 is the whole.
Question for students: (what) of 2/3 is 3/4? Go through similar process as above, resulting in 9/8 of 2/3 is 3/4. Start with two identical whole rectangles. Slice and copy one to make 3/4. Slice and copy the other to make 2/3. Slice and copy the 2/3 rectangle to make the 3/4 rectangle.
Example recording:
9/8 of 2/3 is 3/4. Slice 2/3 into 8 and copy 9 times.
Instruct students working on level 5 to write similar sentences for consecutive pairs of customer orders.

(2) Explicitly mention the connections to multiplying (dividing) fractions and the algorithm for multiplying (dividing) fractions.

Activity 1 Worksheet

Estimate & Compare Activity

Student Name: ____________

Date	**Order #**	**First Guess**	Too Little, Too Big Or Just Right??	**Second Guess**	Too Little, Too Big Or Just Right??	**Third Guess**	Too Little, Too Big Or Just Right??	Correct Order

CandyFactory Prompts and Reflection Questions

The following are prompts and reflection questions that teachers can pose to students playing *CandyFactory*. Some of these questions are best posed for individual students during game play, when they encounter particular problems. Other questions might be posed to the whole class immediately after playing the game, so that they can reflect on what they learned. These prompts and questions are organized by level—the level in which they might be most pertinent.

Level 1:
[when students have completed an order but before shipping] What fraction did you just make? What does that fraction mean?
[also encourage students to consider equivalent fractions] Is there another way you could have done it?

Level 2:
[questions that focus students on the reciprocal relationship between the number of piece and the size of each piece; for example…] Which one is bigger, 1/7 or 1/6?
[after slicing] Why do you think the computer makes all of the pieces equal?

Level 3:
[after seeing the customer order and choosing the whole, ask students to estimate…] What fraction you think it is?
[after measuring an incorrect attempt] Was it too big or too small? What would be a good guess now?
[after shipping an order] What fraction of the whole did you just make? Is there another fraction you can make to satisfy the customer order?

Level 4:
[after seeing the customer order and choosing the whole] Is the fraction bigger than the whole or smaller than the whole? How much bigger than the whole is it?

Level 5:
[after slicing] What fraction is each piece of the given candy bar? What fraction is each of the whole?

In general, reflection questions, such as "What did you just do and why?" can support students' reasoning within the game and help them to transfer that knowledge to situations outside of the game. At the end of a game playing session, students might be asked, "If you were promoted you to manager, what would you tell your employees about how to efficiently satisfy customer orders?" (i.e., "what did you learn from your experience as a factory worker?").

References

McCloskey, A., & Norton, A. (2009). Modeling students' mathematics using Steffe's advanced fractions schemes. *Mathematics Teaching in the Middle School, 15*(1), 44-56.

Norton, A., & McCloskey, A. (2008). Teaching experiments and professional development. *Journal of Mathematics Teacher Education, 11*(4), 285-305.

Steffe, L. P., & Olive, J. (2010). *Children's fractional knowledge*. New York: Springer.

The first two references are articles that appear in NCTM teacher journals; they provide an introduction to Steffe's fractions schemes. The final reference is a book that describes these schemes in more detail.